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Prospective mathematics teachers' extrapolative reasoning about misleading bar graphs

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This study investigates prospective mathematics teachers' (PSTs) reasoning about magnitudes in misleading bar graphs. We report results from three student teachers who worked in a group. They made sense of vertically-oriented bar graphs whose vertical axis was not equi-spaced or y-axis did not start with zero. We analyze PSTs' extrapolative reasoning that refers to the ways they identified implicit relationships in graphs. The results demonstrated that PSTs modified or re-produced the bar graphs to overcome the misleading information, and re-labeled the numerical values of the categorical variables. They focused on arithmetic calculations and used them as a checking tool when they modified the bar graphs. They changed the location of the zero baseline and tended to keep the lengths of the bars invariant.

Keywords: Prospective mathematics teachers, misleading bar graphs, magnitude, extrapolation.

Introduction

Huff (1993) emphasizes that we are often exposed to misleading statistics in daily life. Scaling is one of the most common types of misleading statistics. In most cases, large increments of data display could compress the graph vertically. However, if the increments get smaller, the vertical graph could seem curved or the graph could have zigzags (Huff, 1993). It is also common to put a break in the scale on the vertical axis. This change places the different intervals for the axes to represent the variables, which shows different relationships between the variables other than they actually have. Graphs that incorporate a third dimension are most often confusing for the graph reader, particularly when comparing two or more values. In this sense, the height of neighboring bars affects the accuracy of measuring one value (Huff, 1993).

When students learn to recognize misleading statistics, they develop a new understanding of presenting data. Even in classroom environments and in textbooks, it is possible to encounter misleading or incorrect examples. In particular, statistical studies may be misleading and "at the heart of the matter lurks a black box, the fuzzy and multifaceted 'context' or 'personal factors' or 'subjective meaning', which cast doubt on both simple counts and causal explanations" (Wolf, 2007, p.26).

Bar graphs represent categorical data with rectangular bars that signify measured values. Also, bar graphs show frequency or proportion of categorical data. They highlight relative magnitudes and allow for encoding absolute values of discrete data (Friel, Curcio, & Bright, 2001; Curcio, 2010). Friel, O'Connor, and Mamer (2006) emphasize that "a bar's height is not the value of an individual case but rather the number (frequency) of cases all have that value" (p.125). Although

previous research studies have highlighted students' difficulties reading bar graphs (see Shah & Hoeffner, 2002), there is little research that indicates how students reason about magnitudes in bar graphs with a misleading representation. The purpose of this study is to investigate prospective mathematics teachers' reasoning about magnitudes in bar graphs when they modify or re-produce the graphs to overcome the misleading information. The research question of this study is: "In what ways do prospective mathematics teachers reason about the magnitudes in misleading bar graphs while they go beyond the data?"

Theoretical framework: Graph comprehension

Friel et al. (2001) describe *graph comprehension* as "graph readers' abilities to derive meaning from graphs created by others or themselves" (p.132). The researchers identified three important components of graph comprehension. According to the first component, the graph reader "must understand the conventions of the graph design" (p.152) to be able to obtain information from the graph. For example, "a bar graph highlights relative magnitudes, which are considered analog information; numerical values can be approximated only through scale interpolation" (p.140). In this sense, the graph reader should be able to read the *specifiers* and *labels* on bar graphs. Specifiers are visual representations that signify the data values (e.g., bars on bar graphs) or marks that indicate relationships between the data values on a graph (e.g., the box in a box plot that signifies the interquartile range). Curcio (2010) refers to the level of graph comprehension that "requires a literal reading of the graph" as *reading the data*. The *background* information (coloring, grid, etc.) may help the reader identify the specifiers in the graph (Friel et al., 2001). Also, Friel et al. (2001) refer to the axes, scales, grids and reference markings as the *framework* of a graph.

The second component of graph comprehension emphasizes manipulating "the information read from a graph" (Friel et al., 2001, p.152). Manipulation of the obtained information from specifiers in a graph requires making comparisons (e.g., quantities) and calculations (e.g., addition, division) to combine and integrate the data – that is also known as *interpolation*. For example, one may compare the relative heights or lengths on a vertically- or horizontally-oriented bar graph, and find the proportional relationship between the values of individual specifiers. Curcio (2010) associates this level of graph comprehension with *reading between the data*.

The third component of graph comprehension emphasizes processing the information in the graph taking into consideration the context of the data "to generalize, predict or identify trends" (Friel et al., 2001, p.152) – that is also known as *extrapolation*. In extrapolative reasoning, students read *beyond the data* and find patterns or relations that go beyond values already displayed or known. As Curcio (2010) highlights, "whereas reading between the data might require that the reader make an inference that is based on the data presented in the graph, reading beyond the data requires that the inference be made on the basis of information in the reader's

head, not in the graph" (p.9). Extrapolative reasoning ability may require modifying or re-producing a graph to present and demonstrate new information that is not found in the graph.

Methods

The participants of this research study were selected from a group of prospective middle school mathematics teachers who were enrolled in a methods course about teaching mathematics with technology. The class met two hours per week and the first author was the instructor. The participants were in their third year of the program. One week prior to the study, prospective mathematics teachers (PSTs) learned about making graphs (e.g., bar graphs, histograms, etc.) in Microsoft Excel and GeoGebra. However, the instructions for making graphs aimed to develop PSTs' technological knowledge. The data were collected from nine PSTs (four female, five male) outside the regular class hours. The PSTs were not taught how to make misleading graphs in the computer programs. The participants worked in groups of three. For this study, we report on results from one of the groups. The group members were Nese, Sevgi, and Umut (all names are pseudonyms). Nese and Sevgi are female, and Umut is male.

The participants solved three tasks, and worksheets guided them to investigate vertically-oriented bar graphs. The PSTs were not informed that the graphical displays were misleading. Also, the worksheet questions did not ask the participants to modify or re-produce the bar graphs. For each task, they were given a color-printed worksheet with four additional black and white copies. After the completion of the three tasks, a whole-class discussion was generated. In the first worksheet, the vertical axis of the bar graph was not equi-spaced (see Figure 2a). The graph with some data labels indicated the amount of money a child was given by his family members. The PSTs used the horizontal lines on the graph to identify the missing data labels on the y-axis. We asked them to find the relative ratios of money of the child's family members (e.g., the ratio of money with grandfather and uncle). In the second worksheet, the 2015 PISA scores of three countries (USA: 470, Croatia: 464 and Greece: 454) were represented with a bar graph whose y-axis did not start with zero (see Figure 3). In this task, The PSTs most often reasoned about counter examples. For example, they were asked how they would respond to a child who claimed that the score of Croatia was 6 times of the score of Greece. The third worksheet asked the participants to compare the number of singles a music recording studio sold using the information from a bar graph that incorporated a third dimension. However, in this study, results from the third task were not reported.

While the participants were solving the tasks, the first author monitored them. Also, he became involved in group discussions, gave prompts, and asked follow-up questions. A video camera for each group was placed at a fixed view. Also, the second author videotaped the interactions among the PSTs while the instructor was monitoring their progress. The PSTs' gestures were captured from the video recordings. After the study, the researchers constructed verbatim transcripts coordinated with gestures and written responses of the participants.

In this study, we did not focus on the PSTs' literal reading of the graphs. Rather, we were interested in identifying moments when the PSTs made sense of information that was not provided directly in the graphs. The participants' reasoning about the graphs to obtain information was categorized into two: interpolative and extrapolative. On the one hand, the PSTs' use of specifiers (e.g., horizontal lines on the graphs) to find an unknown value, comparison/manipulation of information in the graphs and arithmetic calculations were identified as their interpolative reasoning. On the other hand, we categorized the PSTs' strategies for rescuing the graphs from misleading information as their explorative reasoning, in particular when they modified or re-produced the graphs. We focused on in what ways they took into consideration the magnitudes of the bar graphs and estimated a value following a pattern in their new data displays. For example, we identified whether they preserved the lengths of the bars when they modified the graphs. The central focus of the results was on the PSTs' extrapolative reasoning in the process in which they rescued the graphs from misleading information. We also analyzed the PSTs' interpolation process to show how they laid the foundations for their explorative reasoning. In other words, the interpolation process was helpful in identifying how they linked their modified/re-produced graphs with their interpolative reasoning to indicate the true relationships between the categorical variables.

Results

In the first task, the graph indicated the amount of money a child was given by his family members. The PSTs found the missing specifiers on the y-axis counting the lines between two known numeral values using them as reference markings using their interpolative reasoning. Then, they focused on the arithmetic calculations as asked in the worksheet. When they were asked if they could identify the relationships between the categorical variables without making any calculations, they considered the magnitudes of the bars. The researcher's prompt encouraged them to modify the graph. For example, Sevgi measured the length of the grandfather's bar placing her thumb and index fingers on the bottom and top of it as shown in Figure 1a. Then, she kept the same distance between her fingers and made a part-to-part comparison between the lengths of bars of uncle and grandfather (Figure 1b). Finally, she figured out that the length of the grandfather was two and a half times of the uncle's. However, in their earlier arithmetic calculation, they found the ratio of money with grandfather and uncle was 3.



(a) (b) (c)

Figure 1: (a) Sevgi takes the length of the bar of the uncle as a reference, (b) She makes a part-to-part comparison between the lengths of the bars of the uncle and the grandfather, (c) She notices

the length of the grandfather is two and a half times of the uncle's

Then, using explorative reasoning, they modified the bar graph to be able to use the lengths of the bars and to check whether the ratios between the categorical variables were consistent with their calculations.

The PSTs equalized the scale of the axis and increased the numeral values of the y-axis by 2. However, this modification gave rise to changing the amount of money given by the family members. For example, the numeral value of the mother changed from 28 to 16. Then, they cut off the bars from the value of 2, and added the cut-outs onto the corresponding bars as shown in Figure 2a. This strategy allowed them to make the intervals increasing by 4. They located the cut-outs onto each bar by adding 2, and preserved the lengths of the bars. Then, they referred to the intervals between the horizontal lines as units and counted them (Figure 2a). Umut said: "Then, if we look at the amount of the money the grandfather and father gave: 1-2-3-4-5-8-9 [pauses]. Isn't it one half [means 9.5]? If we count the father's: 1-2-3-4 and a half. 9.5 and 4.5." Using interpolative reasoning, the PSTs noticed that the re-labeled specifiers (increments) did not match with their arithmetic calculations. The strategy of adding cut-outs onto the corresponding bars did not satisfy the participants. They investigated new strategies to overcome the misleading information.

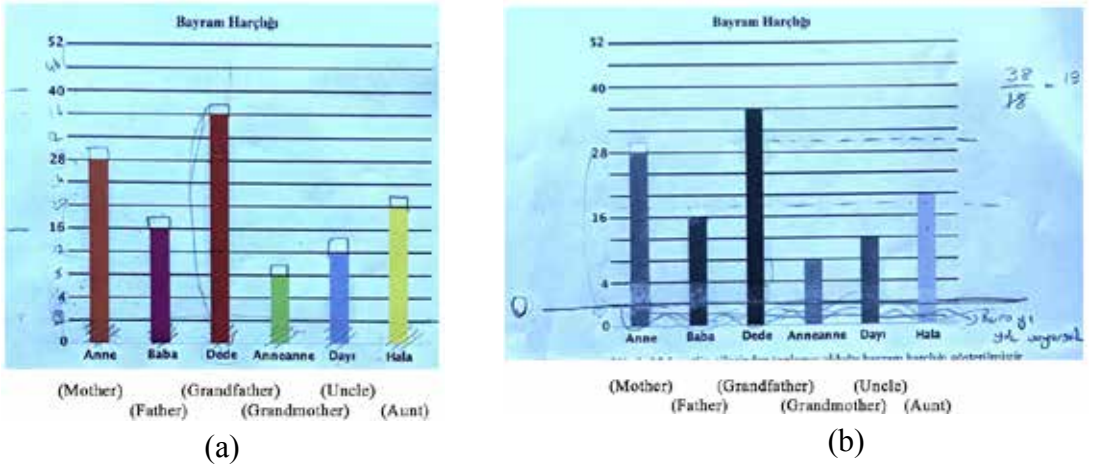


Figure 2: (a) PSTs cut off the bars from the value of 2 and added the cut-out bars onto the corresponding ones, (b) They quit adding the cut-out bars onto the corresponding bars

Nese noticed that if they canceled out the cut-outs (Figure 2b) and counted the intervals between the horizontal lines, the ratios between the categorical variables were the same as those in their earlier calculations. Nese's strategy made the graph equi-spaced and allowed them to count the intervals between the horizontal lines. Then, the PSTs compared the magnitudes of the grandmother and the aunt in their new strategy. For example, in their earlier calculation, they found the ratio of money with aunt and grandmother as 2.5 (20/8), and Nese used this information to support her claim. The below excerpt indicates Nese's approach to the problem.

Nese: For example, let's do not think of that portion [the cut-out] as if it was not there. Let's write zero here and when we count the intervals [between the horizontal lines], we find the same ratios.

Umut: Let's check aunt's and grandmother's. Let me check the aunt's.

Nese: 5 is for the aunt, and 2 for the grandmother.

Umut: 2.5 [the ratio]. Exactly [the same].

Sevgi: Yes. Give me a high five!

In the second task, the participants most often talked about a child's claim that the PISA score of Croatia was six times of the score of Greece. The PSTs read the specifiers and concluded that the claim of the child in the problem context was false. However, Sevgi emphasized that, in the first task, they changed the location of the zero baseline and used the intervals between the horizontal lines. She suggested re-labeling the zero baseline, as it was 452 in the original. In other words, she considered the relative magnitude 452, and subtracted it from the numerical values of the bars. Accordingly, she revised the numeral values of the countries as shown in Figure 3 (USA: 18, Croatia: 12, Greece: 2). Nese and Umut did not agree with Sevgi, and they convinced her by saying:

Umut: We can compare proportionally as they have equal intervals [in the first task], can't we? But, the student [child] is misled here because the graph [y-axis] does not start with zero. It [the child's claim] would count right if this [Figure 3] started with zero…

Sevgi: We removed [the cut-outs] here in this bar graph [refers to the first task]. We removed the same values from all. We can think of it "6" [the ratio of scores with Croatia and Greece] when we take away the same numeral values from all.

Umut: Since it asks for the total score, I think we should not take them away. Especially, we will consider it 470 (for USA) and they [the scores] cannot be reduced.

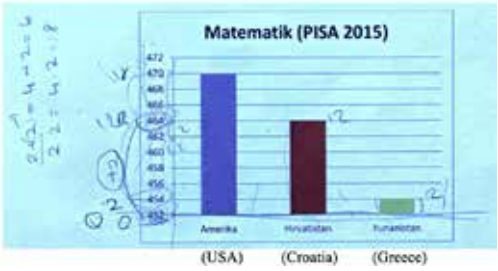
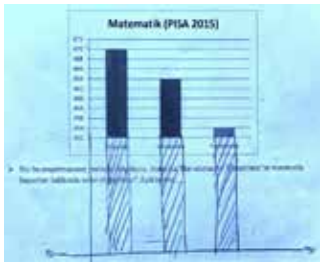


Figure 3: Sevgi re-labels the numeral values of the countries

The PSTs made a decision to start the graph at zero, in order to be able to show the true relationships between the scores of the countries. They extended the magnitudes of the bars to

the zero baseline (Figure 4a). When the instructor became involved in the group discussion, Umut made a gesture as shown in Figure 4b to demonstrate that they extended the zero baseline. He said:

Umut: To able to show the actual magnitudes. The graph shows the score of Greece with an increment of 2, but it actually earned a high score, 452. Namely, there is not much difference [in magnitudes] between 452 and 470. It shows only the upper section [of the bar graph] with an increment of 2. It looks like it [the graph] conceals the actual graph in the background. So, we wanted to make it clear that it [the ratio of scores with Croatia and Greece] is not 6. Eventually, the child numerically sees that 464 is not six times of 454.



(a)



(b)

Figure 4: (a) PSTs extend the magnitudes of the bars to the zero baseline, (b) Umut makes a gesture that signifies the extension of the bars

Concluding remarks and implications

The results of this research suggest that the PSTs focused on performing arithmetic calculations in their early reasoning about the task without considering the graphical displays were misleading. They performed arithmetic calculations to find the proportional relationships between the variables. When the instructor gave a prompt about finding the relationships without making any arithmetic calculations, the PSTs stressed the misleading information. They identified the proportional relationships taking into consideration the lengths of the magnitudes of the bars, and figured out that the relative ratios did not match with their earlier arithmetic calculations. Then, they became more interested in modifying or re-producing the graphs to show the true relationships between the variables. They used their calculations as a checking tool when they modified or re-produced the bar graphs, and re-labeled the numeral values of the variables to overcome the misleading information. The PSTs modified the zero baseline to make the y-axis equi-spaced, cut off the bars from specifiers, and added the cut-outs onto the corresponding bars. They tended to preserve the lengths of the bars invariant when they modified the location of the zero baseline. After they checked their arithmetic calculations, the PSTs noticed that the relationships between the categorical variables were not consistent with their calculations. In this sense, their arithmetical calculations (interpolative reasoning) laid the foundations for their explorative reasoning.

We observed differences in graph comprehension abilities of the participants. For example, Sevgi focused on the magnitudes of the bar graphs when she made an inference. On the other hand, Nese and Umut took into account the arithmetic calculations as a checking tool. As Friel et al. (2001) state, graph comprehension requires spatial judgments or modifications in specifiers and labels in graphs. Future research may indicate how individual differences may affect graph comprehension of students when they make sense of misleading graphs.

One of the limitations of our study was that the PSTs reasoned about vertically-oriented bar graphs. However, the affordances of vertically- and horizontally-oriented bar graphs differ (Shah & Hoeffner, 2002). For example, "horizontally oriented bars may be better for other dimensions for which the horizontal dimension is more meaningful, such as in depicting data about the distance traveled (p.51)". Researchers may investigate how students' making sense of misleading graphs differs in vertically- and horizontally-oriented bar graphs.

It is important to examine not only the axes of bar graphs and check whether they are starting with zero (when graphing positive values) but also if the axes are equi-spaced. If the y-axis is not equi-spaced or does not start with zero, the graph may cause misleading information for the reader. However, a bar graph with a zero baseline may also give rise to missing the relevant information of data. For example, suppose we measure the body temperature of a person with fever in Fahrenheit for a day, at intervals of an hour. If we show the body temperature of the person using a zero baseline in a line graph, it may indicate almost a straight line and conceal the jumps between the neighboring hours. In this sense, the context of the data matters.

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